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LLNL-TR-692417

Method to Compute CT System MTF

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May 18, 2016

This work performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344.

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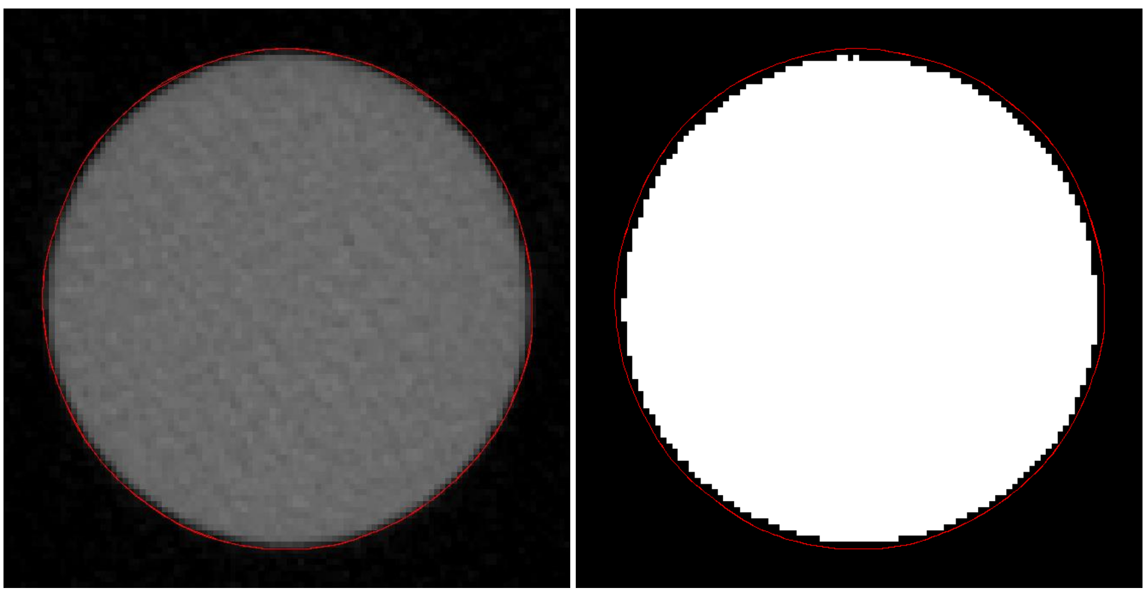
The modulation transfer function (MTF) is the normalized spatial frequency representation of the point spread function (PSF) of the system. Point objects are hard to come by, so typically the PSF is determined by taking the numerical derivative of the system's response to an edge. This is the method we use, and we typically use it with cylindrical objects. Given a cylindrical object, we first put an active contour around it, as shown in Figure 1(a). The active contour lets us know where the boundary of the test object is. We next set a threshold (Figure 1(b)) and determine the center of mass of the above threshold voxels. For the purposes of determining the center of mass, each voxel is weighted identically (not by voxel value).



(a) (b)

Figure 1. The test object is found and (a) an active contour is wrapped around it. We set a threshold (b) and determine the center of mass of the above threshold voxels.

Once the center of mass is determined, an annulus of data about the edge is extracted. An illustration of this annular data set is shown in Figure 2.

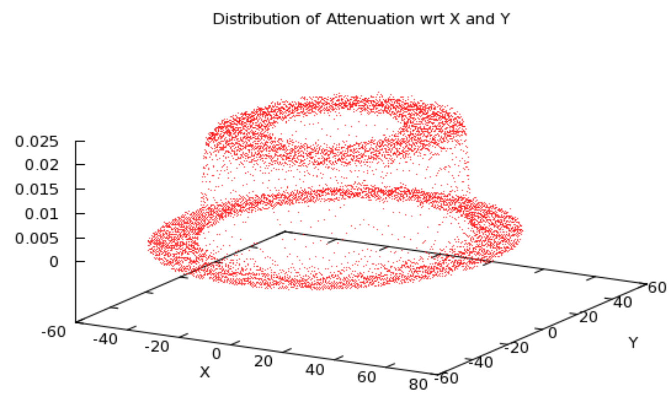


Figure 2. Distribution of attenuation (relative to cylinder center of mass) as a function of x and y.

Next the data are sorted in order of radius (see Figure 3).

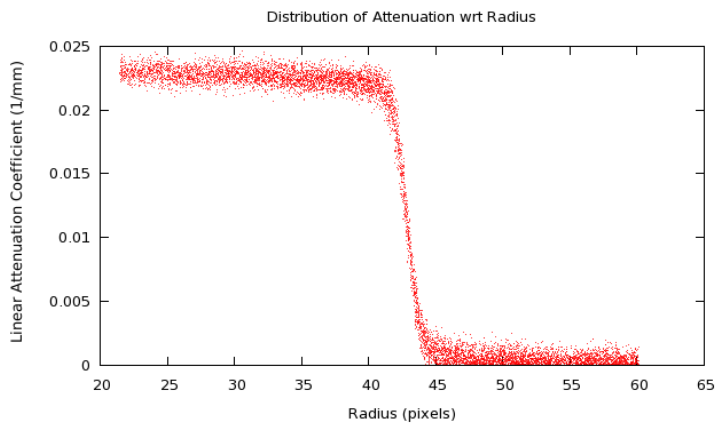


Figure 3. The annular data are sorted in order of radius.

At this point there are two directions that the data processing can proceed along: analytic processing and numerical processing.

For analytic processing we assume that the point spread function is Gaussian. In that case the edge data can be fitted using an error function (erf)[1], as in Figure 4. The functional form that the edge should match is

$$\hat{y} = A + B\,\mathrm{erf}(\,C\,(x - D))$$

where x is the radius, y is the value, $\hat{y}$ is the estimated value and A, B, C, and D are the parameters we are looking for. We search (A, B, C, D) parameter space to minimize the error between y and $\hat{y}$. The derivative of the erf is a Gaussian, so we can analytically determine the Gaussian PSF width from the parameter C by

$$w = \mathrm{abs}(\frac{1}{C})\delta$$

where δ is the voxel spacing and w is the width of the Gaussian. The PSF is of the form

$$PSF = \exp(-x^2/w^2)/(w\sqrt{\pi})$$

[1] M. Abramowitz and I. A. Stegun, Handbook of Mathematical Functions, p. 297, Dover Publications, Inc., New York, 1972.

Once we have the Gaussian PSF we can analytically Fourier transform it to get a Gaussian MTF.

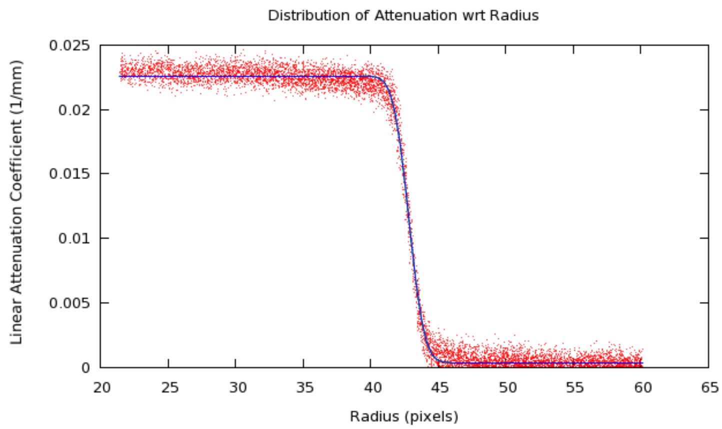


Figure 4. Error function fit to annular data sorted by radius.

For numerical processing, we resample the radial data and use a centered difference to get a numerical derivative, yielding the PSF. We use a fast Fourier transform on the PSF to obtain the MTF.

Figure 5 shows both the numerically and analytically derived MTF from a cylindrical sample obtained on the MicroCT.

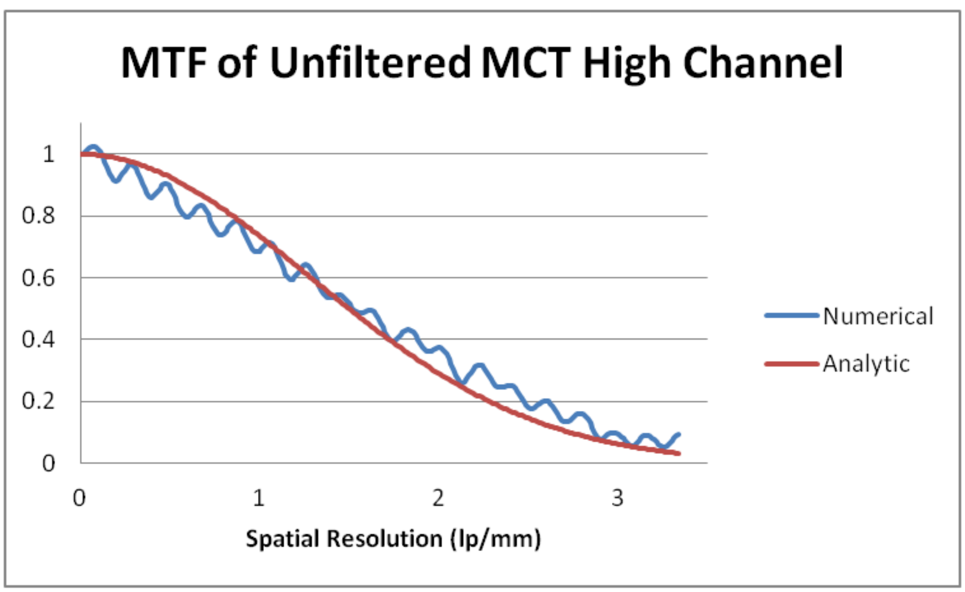


Figure 5. The analytically and numerically derived MTF of a 160kV AlCu MCT cylindrical sample.

In deriving the MTF, there are some requirements on the data. If there is cupping or doming of the data (due to improper beam hardening compensation, for instance) the assumption of a Gaussian point spread function will appear to be violated and the analytically derived MTF will be very different from that of the numerically derived MTF. Examples of these artifacts can be seen in Figure 6 and the effect they have can be seen in Figure 7.

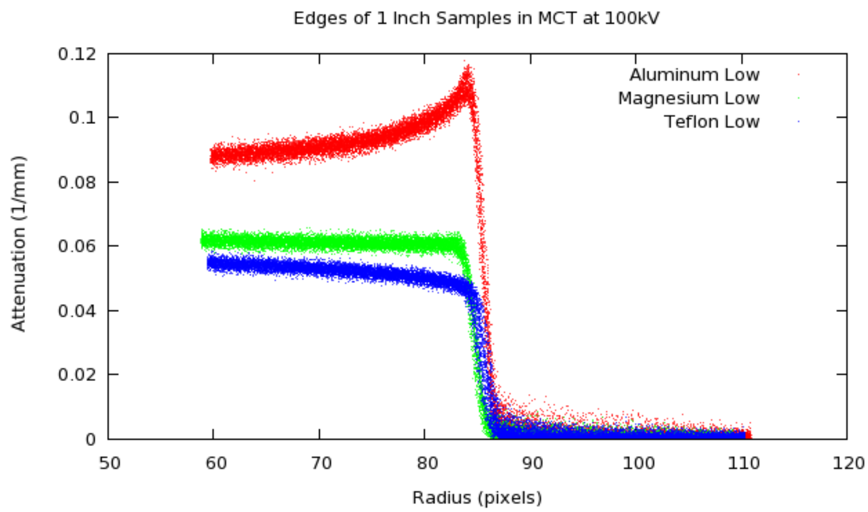


Figure 6. Cupping and doming are evident in the aluminum and Teflon samples, respectively. The magnesium sample has a flat top and is appropriate for use in deriving the MTF.

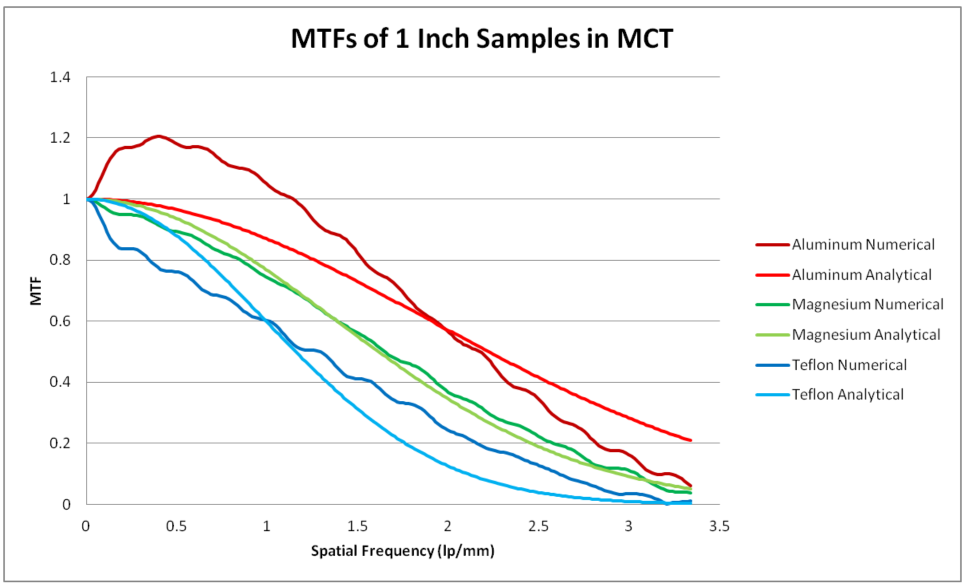
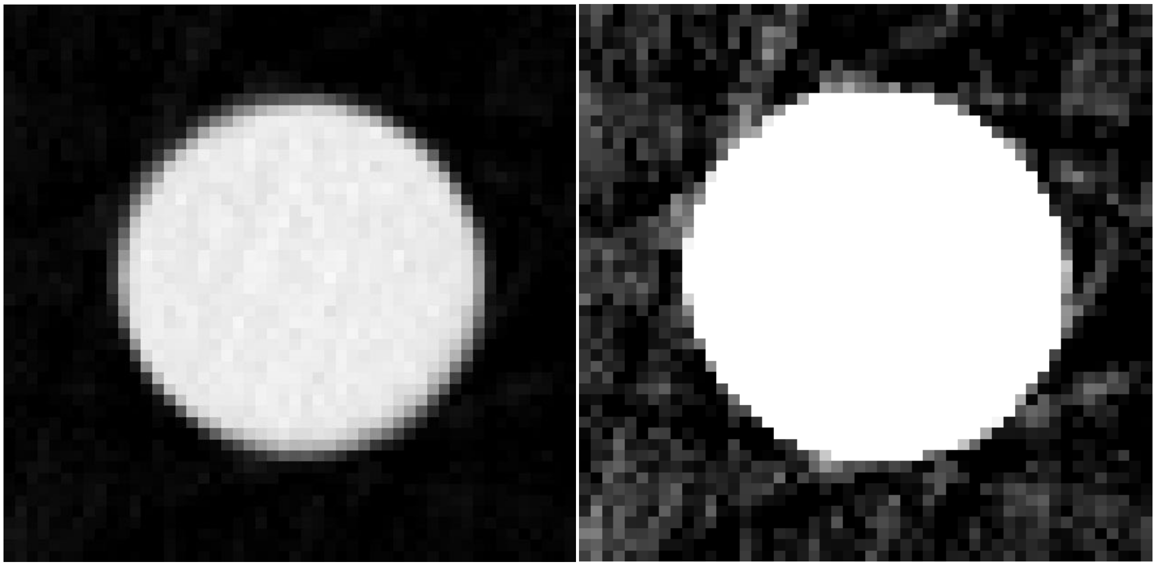


Figure 7. The numerically and analytically derived MTFs of the samples shown in Figure 6.

As we want to compare system performance we use the flat topped magnesium sample. By virtue of the fact that the analytically derived MTFs give no warning of the doming or cupping encountered, we will use numerically derived MTFs for doing comparisons.

Similarly, if there is noise in the reconstruction, it can have a significant effect on the MTF derived. Figure 8 shows the full dynamic range and the low lying noise around the 2 inch diameter Delrin sample in Pelican Case 1. The noise environment is not uniform. This can have a profound effect on the MTF computed. Figure 9 shows the radial distribution of the data, sorted by angular range around the center of mass.



(a) (b)

Figure 8. The full dynamic range image (a) of the Delrin rod does not make apparent the low lying noise seen in (b).

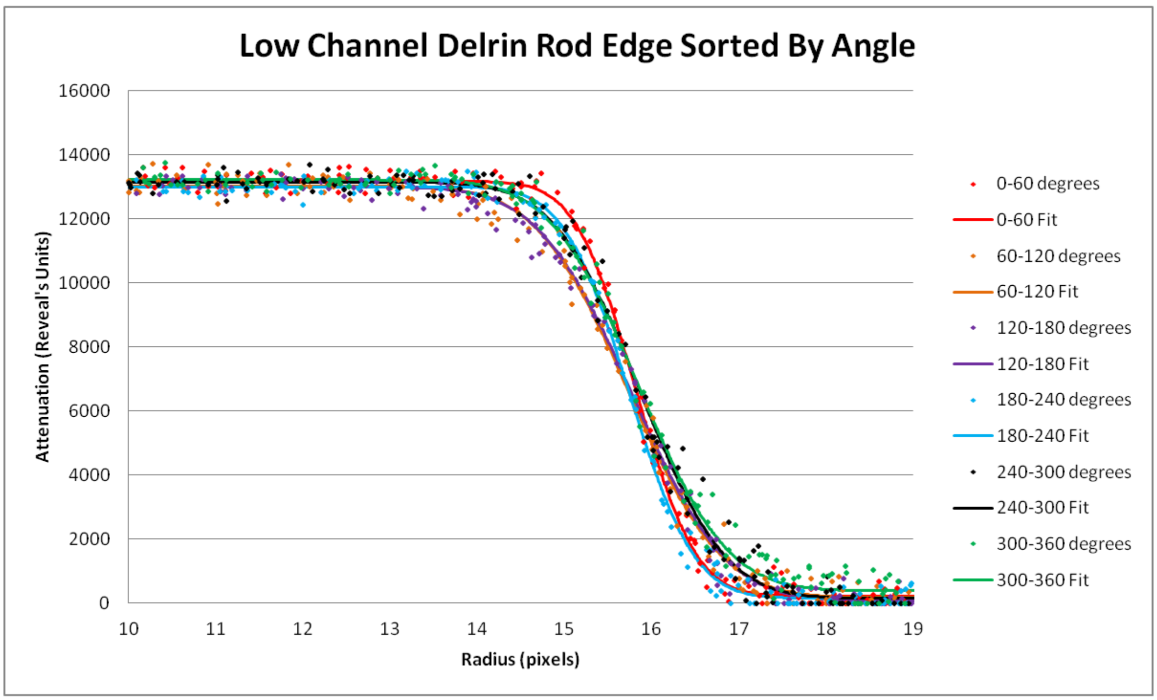


Figure 9. The radial and angular distribution of the data at the edge of the 2 inch Delrin rod in Pelican Case 1.

If we examine the annular data in an angular wedge 60 degrees wide and sweep that wedge around the cylinder, we can analytically derive the PSF as a function of angle. This is shown in Figure 10.

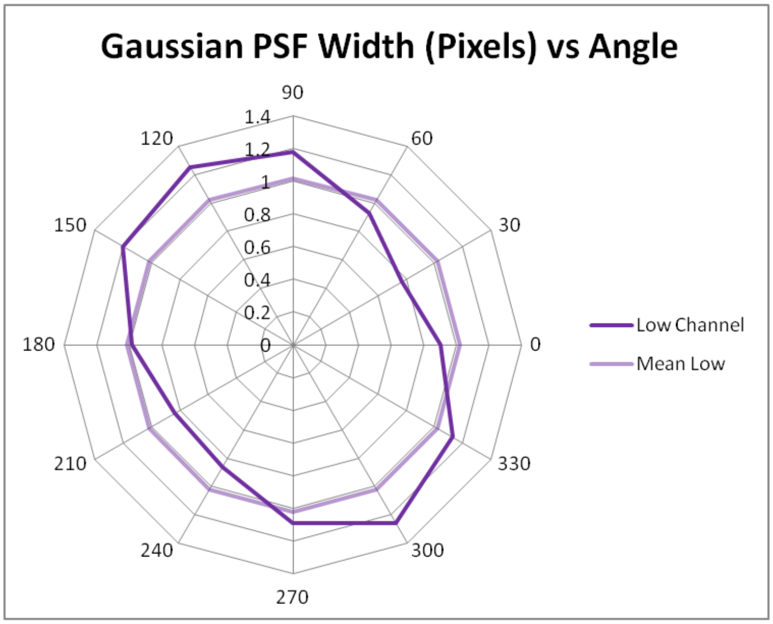


Figure 10. PSF as a function of angle around the cylindrical sample derived using a 60 degree wedge. The noise environment drives this asymmetry. If we use the entire cylindrical sample we get the PSF labeled Mean Low.

Depending on the angular section of data taken, the resultant MTF can change over 30%. In an effort to avoid having to make a choice about the angular size of the wedge, whether the largest or smallest PSF found should be used, etc., we have decided to use the full angular sweep when deriving MTFs.

In conclusion, for the purposes of computing the MTFs we use the full angular range of cylinders which have been reconstructed with flat tops. For the purposes of comparing MTFs, we use the numerically derived MTFs.